The Role of Mathematics in Learning Introductory Quantum Physics

Elina PALMGREN, Inkeri KONTRO, Tommi KOKKONEN
Department of Physics, P.O.B. 64, FIN-00014 University of Helsinki, Finland

Abstract. Quantum mechanics is a notoriously difficult topic to learn. Both the physical interpretation and the mathematical theory behind it are difficult to grasp and many times even in contradiction with our everyday experience. The aim of the research is to study the possible correlations between students' initial level of mathematics (especially that of linear algebra), and the course performance and self-efficacy believes regarding on the topics taught on the introductory course of quantum physics at the University of Helsinki. In this talk we are going to present the data-collection, analysis, and findings so far.

1 Background

Quantum mechanics is a challenging topic to learn, not only because of its conceptual subtleties but also the heavy mathematical machinery behind the physical theory. A student exploring the topic should be able to simultaneously grasp the physical interpretation and the mathematical formulation of the theory, for example [1–2]. Therefore, one main factor in learning quantum mechanics is mathematical proficiency. Especially knowledge of basic linear algebra would be beneficial for understanding the background theory, as the whole quantum theory is based on the notion of vector spaces and their features. Moreover, there is some evidence that learning the necessary mathematical concepts first might be beneficial for learning physics [3].

At the University of Helsinki, the first course of quantum mechanics, *Basics of quantum physics*, introduces basic wave mechanics. The course covers the basics of quantum mechanics from the historical background and old quantum physics up to a qualitative treatment of the hydrogen atom. The course assumes that the students have the necessary mathematical knowledge acquired from other courses, but in practice, students have had problems with using linear algebra (and, to a smaller extent, complex numbers) in physics problems. In previous years, linear algebra has been taught on mathematics courses for physics majors before said students take *Basics of quantum physics.* However, the treatment has been quite perfunctory, as these mathematical skills have not been put to use in physics courses simultaneously.

Some students have also taken linear algebra courses of the Mathematics Department. This causes further divergence in the students' mathematical abilities when entering *Basics of quantum physics.* This spring, for the first time in our physics curriculum, a linear algebra course was taught at the same time with *Basics of quantum physics.* This change was partly done to address aforementioned problems in the physics majors' knowledge of mathematics.

The aim of the research is to study the possible correlations between students' initial understanding of mathematics (especially that of linear algebra), their course performance and self-efficacy believes regarding on the topics taught on *Basics of quantum physics* course.

2 Research

The students taking the course come from various backgrounds: the major subject, year of studies, and initial level vary a lot from student to student. *Basics of quantum physics* is recommended for physics majors in their first or second year of study, depending on study track. Moreover, this spring's course was very apt for studying differing student groups as this was the first time when the linear algebra course was taught at the same time. At the beginning of the course approximately a half of the students on *Basics of quantum physics* course were also enrolled on the linear algebra course, whereas the other half are either older physics majors or physics minors.

To probe students' initial level of mathematics, a pre-test was prepared using STACK (*System for Teaching and Assessment using a Computer algebra Kernel*) system and conducted (N = 73) on the first lecture of *Basics of quantum physics*. The test consisted of five linear algebra problems of different difficulty levels. In addition to knowledge of linear algebra, the test probed familiarity with complex numbers.

At the end of the course, students' self-efficacy beliefs were measured using a questionnaire with multiple-choice questions. The questions were adapted for quantum mechanics from the survey used by Bailey et al. [4] for measuring motivational factors in astronomy.

The final exam contained a problem inspired by the exercise used in the research of Gire and Price [5]. In the problem, students are asked to calculate the energy expectation value for a particle in a square well. Firstly, the expectation value is a central concept in the theory and determining it causes difficulty to many students [6]. Secondly, the problem can be solved using different methods. Here, understanding and using linear algebra should give a student significant advantage.

In this talk we are going to present our data, analysis, and findings so far.

References

[1] C. Singh and E. Marshman, Review of Student Difficulties in Upper-Level Quantum Mechanics, *Phys. Rev. ST Phys. Educ. Res.* 11, 020117, (2015).

[2] B. W. Dreyfus, A. Elby, A. Gupta and E. R. Sohr, Mathematical sense-making in quantum mechanics: An initial peek, *Physical Review Physics Education Research*, v13 n2, (2017).

[3] M. Bassok and K. J. Holyoak, Interdomain transfer between isomorphic topics in algebra and physics. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 15(1), (1989).

[4] J. M. Bailey, D. Lombardi, J. R. Cordova, and G. M. Sinatra, Meeting students halfway: Increasing self-efficacy and promoting knowledge change in astronomy, *Phys. Rev. Phys. Educ. Res.* 13, 020140, (2017).

[5] E. Gire and E. Price, Structural Features of Algebraic Quantum Notations, *Physical Review Special Topics - Physics Education Research*, v11 n2, (2015).

[6] C. Singh and E. Marshman, Student difficulties with determining expectation values in quantum mechanics, *Physics Education Research Conference 2016*, Part of the PER Conference series, Sacramento, CA: July 20-21, 2016, pp. 320-323.