A conceptual introduction to quantum physics using the sum over paths approach and GeoGebra.

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Abstract. Feynman's sum over paths approach constitutes an alternative route for the teaching of introductory quantum physics which is proving more and more promising in recent years. In this talk we will present a collection of GeoGebra simulations published online by the research group at the University of Pavia as a support for teachers willing to experiment such approach with their students. In particular, we concentrate on simulations realizing a sum over paths representation of quantum bound systems, showing how the approach permits to clarify important conceptual features of quantum theory, including the time-energy uncertainty relationship.

1 Introduction

The research group in physics education at the University of Pavia has been working on the teaching of quantum physics in secondary schools since 2013, proposing a reprise and renewal of Feynman's sum over paths approach and testing it with students [1]. The didactic use of Feynman's formulation of quantum physics had been proposed by many authors in the past [2] and educational advantages have been highlighted both in the previous literature and in our own works. By using the representation of complex numbers as "tiny arrows" already suggested in Feynman's QED [3], little advanced mathematics is required, and students can concentrate on the conceptual aspects of the theory. The resulting formal model used is quite simple, and can easily be visualized.

Computer simulations allow to fully exploit the visualization possibilities offered by Feynman's formulation. At a very early stage in our research program, we decided to use the software GeoGebra for integrating our approach with educational simulations. GeoGebra allows to design highly interactive simulations through the use of sliding bars ad checkboxes and is a very widely known and used software in the community of mathematics and physics teachers. Thus, teachers can also work autonomously on the simulations we produced, modifying and adapting them to their educational needs, or creating entirely new ones. The objective of our work is to provide teachers with supporting materials for building teaching-learning sequences with more ambitious goals than simply discussing the initial crisis of classical physics.

In the talk we will discuss some of the simulations which have been included in the online repository in GeoGebra Tube at https://www.geogebra.org/m/Q6waMV2v, concentrating on those allowing a simplified presentation of advanced conceptual aspects of quantum theory.

2 Sum over paths approach to quantum bound systems

One of the most innovative elements in the Pavia proposal is the possibility of discussing, using a unified language, both open and bound quantum systems, *without* introducing the Schrodinger equation. In fact, the "tiny arrow" rotating in space along each path of the quantum object allows to construct the energy dependent propagator, or Green function, from a "source" at position x_i to a "detector" at position x_f , for both types of systems. The Green function $G\left(x_i, x_f, E\right)$ has a natural interpretation as probability amplitude for a quantum object emitted at x_i with energy fixed at E to be found at x_f , and its square modulus is proportional to the probability of such transition from x_i to x_f. By considering all possible paths from source to detector, one can provide an account both of interference effects for open systems (e.g. interference, diffraction) and the existence of a finite number of allowed energy levels for bound systems.

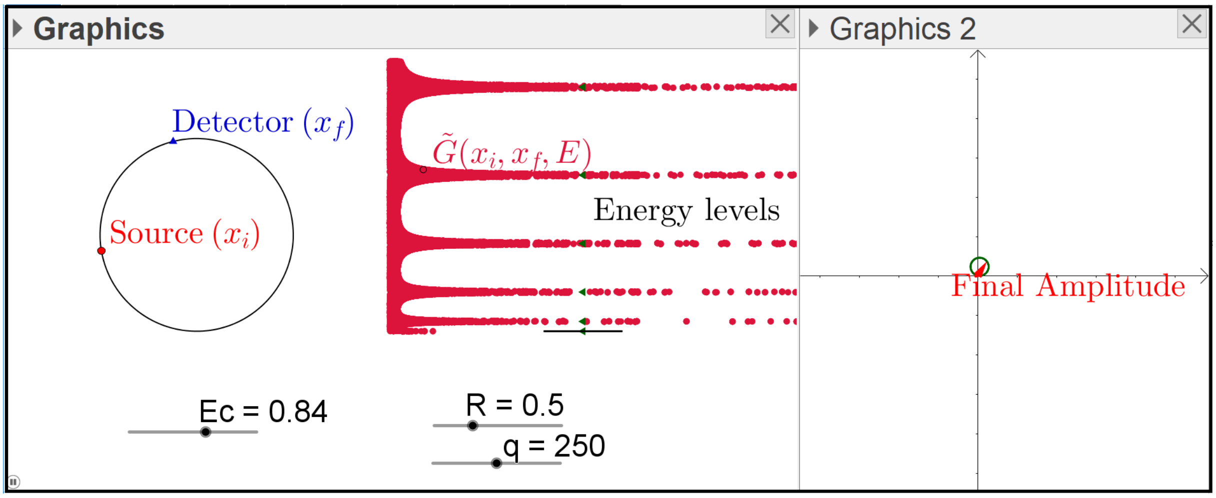

Fig. 1 Sum over paths simulation of a particle confined to a ring

2.1 Indistinguishable paths and the time-energy uncertainty relationships

In modern formulations of sum-over-paths quantum theory, which are especially useful in quantum optics, a particular stress is posed on the idea that, in order to compute detection probabilities, one has to sum the contributions from all experimentally indistinguishable paths from source to detector. When considering quantum bound systems, this idea allows to derive time-energy uncertainty relationships of the form $\Delta t \cdot \Delta E \approx \hbar$ between the finite uncertainty Δt in the emission time quantum objects (which limits the number of indistinguishable paths to be included in the sum) and the width ΔE of the resonance line which, if the number of paths is not infinite, replaces each energy level (see Figure 1). The idea can be developed to constitute a novel educational approach to the time-energy uncertainty relationship, which is notoriously difficult to derive or at an elementary level.

3 Conclusion

The Pavia proposal for teaching introductory quantum physics through the sum over paths approach constitutes a full educational reconstruction [4] of the disciplinary content, which avoids the use of advanced mathematical techniques, and allows to concentrate on conceptual and foundational issues. GeoGebra simulations play a key role in the approach, allowing students and teachers to visualize in a simple way the meaning of the formal model used.

References

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