Derivatives, integrals and vectors in introductory mechanics: the development of a multi-representation test for university students

Marta CARLI, Ornella PANTANO, Mario PERONA, Giuseppe TORMEN

Department of Physics and Astronomy, University of Padova, Via Marzolo 8, 35131, Padova, Italy

Stefania LIPPIELLO

Liceo Statale "A. Veronese", Viale della Vittoria 34, 31034, Montebelluna (TV), Italy

Abstract. This work describes the development, validation and use of a multiple-choice test aimed at investigating students' understanding of derivatives, integrals and vectors in mechanics. The test has been administered to over 1200 introductory physics students at our University. Preliminary results indicate that the test is a reliable instrument and provides interesting insight into students' difficulties in the use of mathematical concepts in physics.

1. Introduction

Difficulties in the use of mathematical concepts and formal reasoning are one of the main obstacles for students entering introductory physics courses at university. A prominent role in understanding a mathematical concept is played by its multiple representations, i.e. the different forms in which it can be expressed, such as graphs, numbers, words, and formal language. Competent use of these representations, including the ability to translate among them, has been recognised as a key competence for successful physics learning [1].

Based on these considerations, we have developed an instrument consisting in a 34-item multiple-choice test. The instrument tests students' competence on selected mathematical concepts and their application to introductory mechanics. The items involve multiple representational formats in order to explore understanding of these concepts more in detail.

2. Instrument design and validation

We started from existing tests in the literature [2-4] and from an analysis of end-of-semester exams. The mathematical concepts covered by the test (derivatives, integrals, and vectors) were selected based on their importance for introductory physics, while the physical content was chosen in order to match the central concepts in an introductory mechanics course.

An initial pool of 78 multiple-choice items was developed. For each mathematical concept, several items were written considering all possible representations both for question and answers. For each mathematics item, a parallel physics item was developed. This draft version of the test was administered to 71 introductory physics students in the second semester of academic year 2016-2017, and item analysis was carried out. Based on these results, several items were deleted or modified and the test was checked by discipline experts for content validity.

The final version of the test contains 34 items (17 mathematical + 17 physical) spanning different combinations of the following representations: graphs (G), numbers (N), words (W) and formal language (F) as described in Fig. 1 (a). This test was administered to introductory physics students enrolled in different degree courses of the faculties of Science and Engineering at the University of Padova. Data collection was carried out at the beginning of the second semester of academic year 2017-2018, after the students have followed introductory calculus courses in the first semester. The data come from a sample of 1235 students.

(a)

Item	FM-1	FM-2	FM-3	FM-4	FM-5	FM-6	FM-7	FM-8	FM-9
Repres.	W/F→W	G→G	G→N	W→W/N	G→W	W→G	G→N	G→G	G→N
Math	Derivative						Integral		
Phys	Position → Velocity						Velocity → Position		

Item	FM-10	FM-11	FM-12	FM-13	FM-14	FM-15	FM-16	FM-17
Repres.	W→G	G→F	G→F	G→F	G→F	G→G	G/F→N/F	G/F→N/F
Math	Vectors							
	Components			Sum		Difference	Dot product	Cross product
Phys	Velocity in 2D		Forces	Forces		Velocity	Work	Torque

(b)

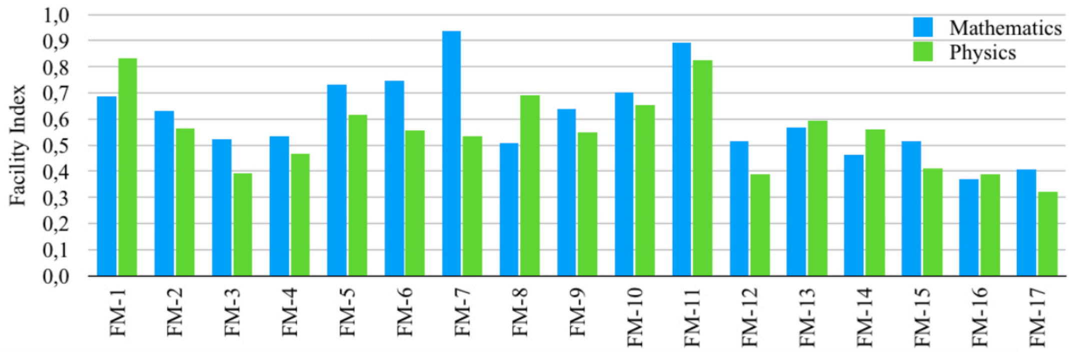

Fig. 1 (a) Item description in terms of content, context (mathematics or physics), and representations. (b) Facility index for the different items. The comparison between parallel mathematical and physical items is highlighted.

3. Results

Data collection and analysis is still ongoing at the time of sending this abstract. Preliminary results suggest that the test is a reliable instrument (Cronbach's alpha=92%). The mean test score was 5,80/10,00. Fig. 1 (b) reports facility indices (i.e. percentage of correct answers) for the items, comparing each mathematical item with its corresponding physical one. Remarkably, quite different results appear for items which are identical in mathematical and physical content, but different in the representational form(s). A more detailed analysis is in progress.

4. Conclusion

We have presented a valid and reliable instrument which allows to characterise students' understanding of relevant mathematical concepts in introductory physics. These results may suggest reinforcement strategies based on the competent use of multiple representations.

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