A ‘misleading mathematical legitimacy’? Analysing cases of critical passivity in teachers

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Abstract. This paper examines a case that seems to block physics teachers’ critical analysis. It concerns explanations that use well-known relationships between physical quantities and derive from them, through correct calculation, an accurate response to the problem, although the proposed modelling of the situation is defective. Illustrative excerpts of a preliminary interview and small group discussions with beginning teachers are analysed and discussed and confirm the difficulty of adopting a critical stance regarding misleading texts of that kind. The paper concludes with a discussion of objectives for teacher preparation and future research.

1 Introduction

In a context where critical thinking is universally advocated, one crucial question concerns the links between conceptual and critical development. In the particular domain of text analysis in physics, previous investigations have reported a co-occurrence in some cases [1] between conceptual expertise in relation to a given topic and critical passivity on encountering a contestable explanation of that same topic. In order to further document this phenomenon of ‘expert anaesthesia’, we selected two cases in which calculations and results were accurate while the modelling process incorporated a serious fallacy. The two examples [1] [2] raise the question of whether a kind of misleading ‘mathematical legitimacy’ might engender the critical passivity observed in teachers when modelling the case situation. To document this question, we used a new situation concerning a solved exercise in elementary thermodynamics. Symptomatic excerpts from a preliminary interview and small group discussions with beginning teachers will be analysed and discussed in terms of possible factors favouring critical passivity. The ultimate goal of this still highly exploratory investigation is to inform a discussion of teacher preparation and the future research agenda in relation to critical passivity and, more particularly, expert anaesthesia.

2 Previous research and research questions

In a previous investigation [1], the phenomenon of *expert anaesthesia* was observed à propos of current exercises presenting hot air balloons as isobaric situations: an hypothesis contrary to the law of fluid statics. This case is emblematic, because this hypothesis leads to a quite acceptable result, via Archimedes’ theorem. More recently [2], we investigated a similar case, concerning a contestable explanation based on accurate calculation and conclusion: it concerns the angle of contact between liquid and solid in the presence of gas. Current explanations rely on what seems to be a free-body diagram of forces acting on an immaterial line, which is meaningless. However, this treatment leads to the correct “Young’s formula”. In this case as with the “isobaric” hot air balloon, the favour of a ritual explanation in the textbook writers and the quasi unanimous lack of critique in teachers are very striking. Given these findings, we thought it worth exploring further the possible influence of the mathematical accuracy of the calculation, combined with the exactness of the result on the absence of critique observed in teachers. Our research questions are therefore: When presenting teachers with that kind of text, shall we observe, as expected, an absence of critique? In such a case, can we access to some reasons for their critical passivity?

3 A misleading calculation about an irreversible expansion of an ideal gas: preliminary findings

In a solved exercise about the irreversible expansion of one mole of an ideal gas from an initial pressure P_A to another P_B ($P_A > P_B$, where P_B = constant value of external pressure) "at constant temperature T", a few lines of calculation were used to calculate the work $W_{irrév}$ done on the gas:

$$V=RT/P \quad \text{(one mole; R: constant of ideal gas)} \tag{1}$$

$$dW_{irrév} = - P_B\, dV = P_B\, RT\, dP/P^2 \tag{2}$$

$$W_{irrév} = \int_{PA}^{PB} P_B\, RT\, (dP/P^2) = RT\, P_B\, [\, -1/P\,]_{PA}^{PB} = RT\, (P_B/P_A - 1) \tag{3}$$

This text was found in a booklet of solved exercises for students in first year at university. There is no error in the calculation, and the result is accurate. Yet the transformation is modelled using a relationship between volume, temperature and pressure (i.e. the ideal gas law) that holds only for states of equilibrium. Note that whatever V(P) relationship is used here, the integral $\int_{PA}^{PB} P_B\, (-\, dV/dP)\, dP$ would be equal to $P_B\, (V_A - V_B)$. Given, then, that the ideal gas relationship holds for the initial and final states, we would obtain the correct result. The important point is that the very ideas of an equation of state and defined values of internal temperature and pressure during this transformation are meaningless. In short, the modelling of this situation is clearly defective.

Following an exploratory interview (which was audio-recorded), the text was submitted to three groups of beginning physics teachers at university (three per group). The participants were told that the calculation and result were accurate, and they were also presented with the correct calculation of the work $W_{rév}$ done on the gas in the case of a reversible expansion. (This time, the modelling is correct: the ideal gas law holds throughout the transformation). They were then asked the following questions. 'Are you satisfied with this explanation, or would you change or add something? Why? Their discussions (which lasted about 20 minutes) were audio-recorded. No participant questioned the use of the perfect gas law to model this transformation. All subsequently mentioned that they were blocked from critiquing the text by the fact that calculation and result were accurate. Excerpts from this interview and these discussions illustrate this phenomenon in more detail.

4 Discussion and perspectives

These preliminary findings illustrate a significant difficulty in critiquing a seemingly correct mathematical approach that is 'guaranteed' by an accurate result but stems from inappropriate modelling of the situation addressed. Contrary to the ritual explanations mentioned above, our last example concerns a case not frequently observed in current teaching environments. Therefore, among the factors likely to have prompted critical passivity, this 'misleading mathematical legitimacy' may intervene independent of the more or less ritual nature of the explanation. In this regard, it would be interesting to extend the investigation to more experienced teachers. The examples discussed here might be used in teacher education to illustrate the distance between obtaining an accurate result and employing valid modelling. More research is needed to inform the design of the appropriate environments.

References

[1] L. Viennot, Teaching rituals and students' intellectual satisfaction, *Physics Education* **41** (2006) 400-408.

[2] L. Viennot, Ascension capillaire : quand le verre semble « hisser » le liquide, *Bulletin de l'Union des Physiciens* **977** (2015), 1201-1212.