Percolation theory for students – a simplified experimental setup

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Abstract. One of the most universal theories, widely used in many areas, is the percolation theory, which describes various transport phenomena in connected clusters in random networks. However, the number of quantitative experiments which allow to familiarize students with percolation is quite limited. This contribution presents a percolation activity based on RC circuit measurements of water network percolation in samples of sand during their free dehydration to the air. Results show the coherency of the measurements taken in a simplified experimental setup with the measurements taken by a professional impedance analyzer. Simplified equipment gives an opportunity to introduce a learning unit on percolation in high schools.

1 Introduction

Percolation theory describes various types of transport phenomena in connected clusters in random networks. Due to its universality, it was widely used for many applications, like for example phase transitions, coffee percolation or neuron activity, nevertheless percolation most often is employed to describe the movement and filtering of fluids through porous materials.

The basic model of percolation is formulated on a lattice, at which every single square can be in one of two states, "occupied" or "empty" [1]. Each site is occupied independently of its neighbors with probability p. If two occupied sites are linked by a path of nearest-neighbor bonds they belong to the same cluster. For every network and transport phenomenon, one can define a critical value p_c, called percolation threshold. For occupation probabilities $p > p_c$ an infinite cluster of the occupied bonds is formed.

As mentioned above, percolation theory is widely used in many scientific disciplines. For example in economy it is use to predict bank smashes [2], in informatics - to analyze cascading events [3]; it can be also used to describe cracking of the trees or epidemic spreading [4]. Despite wide range of applications, percolation theory is not commonly taught, even at the universities. This is a general problem already raised in [5], that advanced or novel physics topics are not included in school curricula, which can be caused among others by the lack of simple experiments necessary to introduce such topics. To fill this gap, we prepared a percolation activity, which could be implemented in laboratories for undergraduate biophysics and nanotechnology students as well as in more advanced high school classes.

2 Percolation activity

We studied conductivity percolation in dehydrating materials, in which electric properties of a sample continuously change with decreasing mass of water in time [6]. In such case, percolation can be described by the equation [7]:

$$(\varepsilon'' - \varepsilon''^{*}) \sim (t^{*} - t)^{\mu};\ t \leq t^{*}$$

where ε'' is the imaginary part of the permittivity, and values with (*) relate to the values at percolation threshold. μ is the critical exponent, characteristic for dimensionality of percolating network [1].

Usually conductivity studies are performed by utilizing impedance analyzer, which usually is inaccessible in high school laboratories or even laboratories for undergraduate students. To familiarize students with water network percolation a simplify experimental setup, based on the RC circuit, was designed. It consists of a parallel plate capacitor filled with the moisture material, connected in series with adjustable resistor and AC generator. Experimental setup is presented in Figure 1. Upper capacitor plate has got holes, which allow free dehydration of measured material to the air. The whole circuit is connected to the oscilloscope, which measures: voltage amplitude on the AC generator (U_{AC}), voltage amplitude on the resistor (U_R) and the phase shift between both signals (φ). Simple equations related to voltage divider and RC phasors enable to calculate the electrical conductance (G) of the capacitor (which changes during the measurements as the effect of decreasing amount of water in the sample), and therefore the imaginary part of the permittivity ε''.

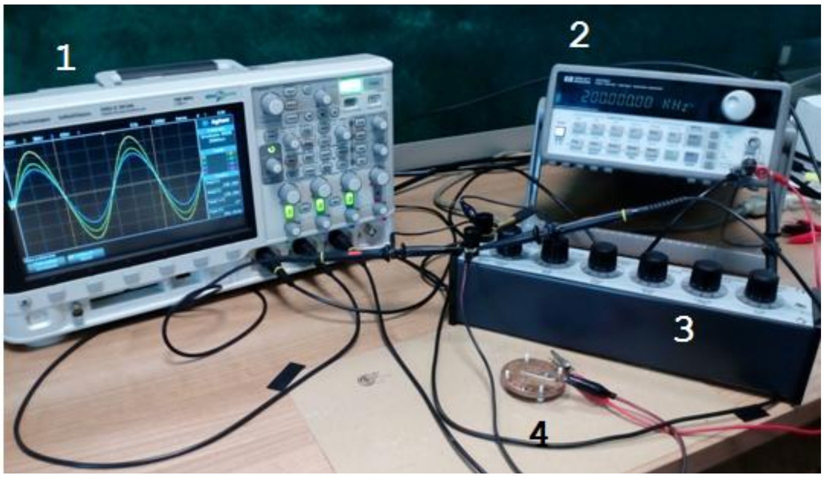

Figure 1: Experimental setup. 1 - osciloscope. 2 - AC generator. 3 – adjustable resistor. 4 - capacitor with a sample inside.

3 Conclusions

Preliminary results have shown that the values of percolation threshold determined from measurements in a simplified experimental setup on samples of sands with controlled diameter of grains correspond to analogues values measured by advanced laboratory equipment (impedance analyzer). On the poster the simplified experimental setup will be presented together with the experimental results as well as a proposal of a learning unit on percolation for students.

References

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