Students' Use and Understanding of Integration in Intermediate Electromagnetism

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Abstract. This presentation reports on a study of how students use and think about integration in an intermediate level electromagnetism course. We found that on entering the course students primarily see integration as a process of evaluation, and that the majority of students have no conceptual aspect in their concept image of integration. By the end of the course the vast majority of students recognize the need to integrate in their description for how they would calculate quantities like charge given a general non-uniform charge distribution. However, only 35% used integration in their calculation of charge given a varying charge density expression.

1 Introduction

In physics, mathematics is one of the foremost ways in which models are communicated. It allows us to apply physics concepts to explain phenomena and also to develop new understandings of the world. As students move through a physics major they are faced with increasingly complex physics concepts that require the use of sophisticated mathematical tools. In recent years, identifying challenges students have using mathematical tools in different physics contexts has been an area of focus for the physics education research community [1-3].

Integration is perhaps the foremost mathematical tool required in intermediate level electromagnetism courses. Research has identified cues that students use to know when integration is required when solving problems [4] and difficulties that students encounter when setting up and evaluating integrals [5]. In a previous study [6], we investigated our students' views of integration before they started their electromagnetism course. We found that when students were asked to interpret the context-free integrals $\int_a^b dx$ and $\int_a^b n(x)\mathrm{d}x$, more than 60% attempted to evaluate the integral (despite it being impossible to do so in the case of the latter), while only 5% mention summation.

In this presentation, we extend this work by looking at the research questions: How do students describe the calculation of quantities like charge given a non-uniform distribution? How do students' descriptions compare to their calculation of the same quantity?

2 Research Context

Our intermediate electromagnetism course covers elements from a standard calculus based introductory electromagnetism course and the first 5 chapters of Griffiths' textbook on Electromagnetism. The course is taken by a diverse group of students: second-year engineering and physics students, as well as fourth-year pre-service high school science and mathematics teachers. The course uses a system of one lecture and two small-group guided-inquiry tutorials per week (one conceptual and one mathematical).

Very early in the semester students work through a tutorial that aims to guide them through the building of an integral, using it as a summation. It follows the Von Korff and Rebello framework [7] in the context of a rod with a continuously varying charge distribution. A similar approach is used repeatedly throughout the semester in other tutorials to calculate quantities such as finding the electric field at a point a distance above the left end of a rod.

Data for this presentation comes from 56 student responses to two questions on an end of semester exam.

3 Calculating Charge

3.1 Exam Questions

The first question asked students to consider a very thin non-uniformly charged rod and describe how you would calculate the total charge on the rod. They were prompted to include a discussion of the mathematical techniques they would use and explain why it is appropriate and/or necessary to use them.

The second question involved a circular disk of radius R and negligible thickness with a surface charge density given by $\sigma = \alpha r^2$. Students were asked to calculate the total charge on the disk in terms of the given quantities and to explain and show their work.

3.2 Results

For the description of how to calculate the total charge on the rod, 86% of students stated that integration would be needed because the rod is non-uniformly charged: 66% had a fully correct and complete description and another 20% had incomplete descriptions (not including how to find the charge on an infinitesimal piece of the rod). The remaining 14% of students stated that total charge on the rod could be found by multiplying the charge density by the length of the rod.

For the calculation of total charge on the disk, only 36% of students used integration. 59% of students started with the expression $Q = \sigma A$ and substituted the given expression for σ. It is possible that many of these students, especially those who included integration in their descriptions, do not recognize $\sigma = \alpha r^2$ as a non-uniform charge density. Students similarly gave very different answers to analogous questions on the electric field, potential, and flux sue to a non-uniformly charged rod.

4 Conclusion

We have investigated how students describe the calculation of charge given a non-uniform distribution, and found striking differences between those descriptions and their calculations.

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