dx versus Δx – more than a translation to Greek

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Abstract. Differential calculus is necessary for the description of more complex physical phenomena. However, mathematics education has shown that many students lack a deep understanding of the concepts of differentials and derivatives. To gain deeper insight into student understanding of these concepts in physics, we developed a questionnaire probing student ideas on instantaneous velocity and electric field (derivative of potential with respect to position) in both graphical and symbolical representations. Many students from four different universities incorrectly refer to a particular case (e.g. E-field of a point charge) or confuse average and instantaneous velocity or electric potential and field.

1 Introduction & problem statement

Differential calculus is needed in the description and analysis of more complex physical problems, but many students lack understanding of the basic concepts of calculus, such as differentials and derivatives [1]. Our study focusses on student understanding of the concept of derivative in physics contexts. Expressions such as $v = dx/dt$, $dW = F\ dx$, and $E_r = -\ dV/dr$ are present in the theoretical description of physics phenomena and in problems related to these phenomena. In this contribution, we report on student answers in questions probing for their understanding of instantaneous velocity and electrical field (as the derivative of potential with respect to position) in a graphical representation.

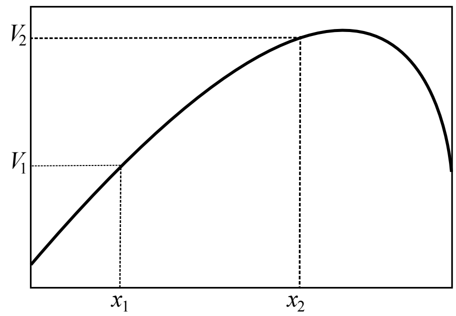
2 Methodology

To gain deeper insight into student understanding of differentials and derivatives in physics, we have developed several open-ended questions in which student ideas on instantaneous velocity and on electrical field (as the derivative of potential with respect to position) are probed both in graphical and symbolical representations.

In this contribution, we focus on two graphical questions: one in the context of 1D kinematics, and one in the context of electrostatics. The questions are given in Figure 1. In both cases, the dependent variable changes non-linearly as function of the independent variable, meaning that instantaneous velocity (or local field) is not the same as average velocity (or field).

The questions were given to cohorts of students in four different universities. A categorization scheme was devised, bottom-up, i.e., from the data.

The figure shows the electric potential V of a particle at different positions x.
Explain how you would calculate the x-component of the electric field at positions x_1 and x_2.



The figure shows the position x of a particle at different times t.

Explain how you would calculate the instantaneous velocity at times t_1 and t_2.

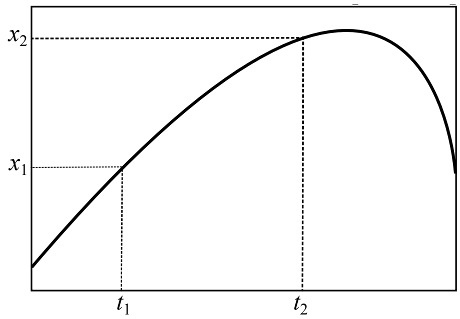


Fig. 1. Questions probing student understanding of derivatives in different physics contexts.

3 Results

The final categorization scheme is given in Table 1. It is remarkable that we could design a set of categories that applies for both contexts.

Table 2. Coding scheme for responses to the questions of Fig. 1.

A. Correct understanding of derivative A1. Answer based on formula $v = \mathrm{d}x/\mathrm{d}t$ A2. Answer based on slope of the tangent in point (x_1, t_1) (not necessarily mentioning the formula)	**A. Correct understanding of derivative** A1. Answer based on formula $E = -\mathrm{d}V/\mathrm{d}x$ A2. Answer based on slope of the tangent in point (V_1, x_1) (not necessarily mentioning the formula)
B. Inappropriate use of uniformly accelerated straight line motion formulas	B. Inappropriate use of a particular electrostatic case (e.g. point charge)
C. Confusion between instantaneous and average velocity	C. Confusion between local and average field
D. Incoherent	D. Incoherent
E. no answer – no explanation	E. no answer – no explanation

4 Conclusion

We have designed open-ended questions probing student understanding of derivatives in case of non-linear relationships between variables. Preliminary results show that a lot of students not clearly distinguish between instantaneous and average quantities or rely on formulas of particular cases that not apply to the problem statement. In the presentation we will describe student results in more detail.

References

[1] Martinez-Torregrosa, R. Lopez-Gay and A. Gras-Marti, Mathematics in Physics Education: Scanning Historical Evolution of the Differential to Find a More Appropriate Model for Teaching Differential Calculus in Physics, *Science and Education* **15** (2006) 447–462.