Linking concepts of linear relations in physics and mathematics

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Abstract. To study student understanding of concepts related to linear functions in physics (kinematics) and mathematics in the 9th grade, we designed and administered a 24-item test in which students must identify or compare the *y*-intercept or the slope in graphs and formulae. Student explanations are categorized and common errors are identified. We focus on the qualitative side of student strategies (such as the use of a dimension analysis in physics) and common errors (such as switching the *y*-intercept for the *x*-intercept and omitting the negative sign of the velocity in graphs and equations).

1 Introduction

Using mathematical concepts to describe physical phenomena is important in physics, even for high school students. Literature however shows that combining physics and mathematics is challenging for students and therefore, the study of student difficulties linking both fields is a hot topic in Physics Education Research [1]. The newly designed integrated STEM (Science, Technology, Engineering and Mathematics) curriculum for secondary education developed in the STEM@school project in Flanders (Belgium), explicitly aims to better link concepts from all STEM fields. In the context of that project, we studied student understanding of concepts related to linear relations both in the context of physics (uniform linear motion) and mathematics in the 9**th** grade. We focus on linear relations because of the known difficulties students have with their abstract representations as an equation or a graph, the latter particularly in kinematics. Furthermore a performance gap has been shown to exist for student understanding of linear relations between mathematics with and without context [2]. Our main research questions are: How do students understand the concepts of 'slope' and 'y-intercept' in a purely mathematical question or a 1D kinematics question? How do students understand the concepts of 'slope' and 'y-intercept' in a graphical or symbolic representation?

2 Method

We designed a test consisting of 12 physics and 12 mathematics open-ended questions. Each item is formulated using a symbolical (formula) or a graphical representation. The problems ask to determine slope (velocity) or y-intercept (initial position) of a given linear relation, or to compare slope (velocity) and y-intercept (initial position) between two given relations. Both positive and negative slopes but only positive y-intercepts are included. A first categorization scheme describing student motivations was build bottom-up from the data (253 students in 7 schools), by the main author. To achieve a sound categorization, the scheme was

– based on a subset of the data – refined by a second researcher and then independently used to analyse a new subset of the data by the first author and a colleague.

3 Results

Table 1 shows the categorization scheme consisting of 5 shared categories for "*y*-intercept" and "slope", 2 which are unique to the "*y*-intercept" and 4 which are unique to the "slope".

Table 1. Concise categorization scheme.

***y*-intercept / initial position**	**Slope / velocity**
Identification through the location of a coefficient in an equation.	
Identification of the root (*x*-intercept). (graphical or symbolical)	
Change representation: Construct an equation.	
Change representation: Construct a graph.	
Change representation: Construct a table.	
Identification of the intersection with the vertical axis.	Reasoning with the "steepness" of a graph.
Calculation of $f(x)$. (often with $x=0$ or $x=1$)	Drawing a triangle on a graph.
	Calculation/comparison of the ratio of differences.
	Calculation/comparison of the ratio of some numbers. (e.g.: coefficients in the equation, *y*-intercept over *x*-intercept, etc.)

Firstly, the scheme achieved a good overall inter-rater reliability (Cohen's Kappa) of 0.61 with item specific values often far higher. Some remarkable findings of our analysis are:

- A minority of students switches representation. Most often, an equation is constructed when a graph is given, indicating higher reliance or pressure on/for equations.
- Quite some students switch *x*- and *y*-intercept in the context of mathematics.
- Comparison and identification of negative slope(s) is more difficult in physics.
- In physics, students often base their motivation on dimension analysis, i.e. they look for "meters over seconds". They select the coefficients they think have these units or (incorrectly) manipulate the equation to end up with an expression for a quantity in meters and for a quantity in seconds and then take the ratio as shown in Figure 1.

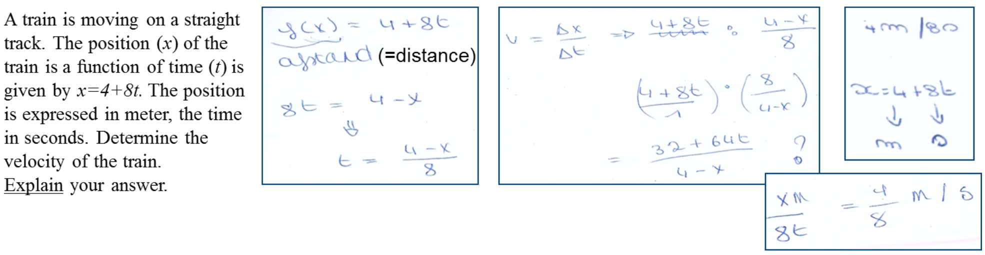

Fig. 1 Question from the test and three explanations from different students.

4 Conclusion

Our results highlight intriguing difficulties and strategies which can be useful for future improvements to teaching approaches. The presentation will discuss the categorization scheme in depth and provide more details about the prevalences of the strategies and errors.

References

[1] M. Planinic, L. Ivanjek, A. Susac and Z. Milin-Sipus, Comparison of university students' understanding of graphs in different contexts. *Phys. Rev. ST Phys. Educ. Res.*, **9** (2013), 020103.

[2] R. Karam (Ed), Thematic Issue: The Interplay of Physics and Mathematics: Historical, Philosophical and Pedagogical Considerations, Science & Educ., **24(5-6)** (2015), 487-805.