Investigating students' difficulties with differential equations in physics

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1 Introduction & problem statement

There are numerous cases in physics where the value of a quantity and changes in that quantity are related. For example, the speed of an object depends on its acceleration; the radioactivity of a sample depends on the amount of the sample present. Except in highly idealized settings, the analysis of these cases requires students to recognize, set up, and solve an *ordinary differential equation* (ODE).

In many universities, including DCU, ODEs are studied in mathematics before they are applied in physics. However, the aims of mathematicians and physicists can be very different. Mathematics modules tend to emphasize the classification of ODEs and their theoretical aspects (questions of existence and uniqueness of solutions). Techniques for solving ODEs are also studied extensively, and typically students acquired a good instrumental understanding [1] of solving ODEs. However, in physics modules, modelling is often emphasized: students must apply mathematical knowledge to interpret a setting, recognize the need for and set up an ODE, solve it, and interpret the solution. Other potential reasons why ODEs present a problem for physics students include gaps in students' mathematical knowledge, and conceptual issues with ODEs.

2 Research questions

This project is a multi-stage investigation that began by identifying the issues experienced by physics students during their study of ODEs before addressing them through the design and implementation of a set of tutorials. The research questions for the project are as follows

1. Do our students have the necessary instrumental understanding in the following areas to succeed in their study of ODEs?
    a. Manipulation of exponentials in equations;
    b. Evaluating indefinite integrals.
2. Do our students have a well-developed concept image of ODEs upon completion of this module?
    a. What is brought to mind when presented with an ODE?
    b. What do students know about ODEs and their applications?
    c. Do they understand what a solution to an ODE is?
3. Has the intervention benefitted our students when learning ODEs?
    a. How does the instrumental understanding of students who experienced the intervention compare to those who completed the module prior to its implementation?
    b. In what way has the concept image of students who experienced the intervention grown?
    c. How is the intervention viewed by the participants?

3 Results

Having surveyed a cohort of physics students who completed a typical service module on ODEs, we found that many of them possessed a fragmented concept image [2] of ODEs and insufficient instrumental understanding to succeed in physics modules involving ODEs. To improve the situation, we designed a set of fourteen tutorials that sought to address these issues. In addition to targeting specific issues with their instrumental understanding (primarily the manipulation of exponents and indefinite integration), the intervention focused on broadening their concept image of ODEs. Tutorials were designed on the meaning of the derivative, direction fields, and the solution nature of an ODE among other things.

The effectiveness of the tutorials was measured using immediate pre/post-testing. In addition to the data from this, delayed post-tests and interviews with students revealed significant gains in the understanding of ODEs as well as an appreciation of the guided-inquiry approach employed throughout the tutorials.

4 Conclusion

While our students have some gaps in their instrumental understanding of their mathematical toolkit, they are hampered much more by an incomplete concept image of ODEs [3]. By studying the conceptual difficulties of physics students with ODEs and carefully designing a guided teaching-learning sequence, we have been able to improve students' conceptual understanding of ODEs without impacting negatively on their instrumental understanding in a manner they appreciated.

References

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