Embedding Diagrams – a Hands-on Activity for Understanding Spatial Curvature

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Abstract. Even the basics of General Relativity are considered difficult for students due to its abstract nature and the lack of concrete demonstrations suitable for a typical classroom. One area that offers some possibility of hands-on activities for students are embedding diagrams, the curved surfaces that illustrate spatial curvature. Plastic models of these surfaces can easily be made using a 3D printer, which is a piece of equipment that is rapidly becoming available to schools and students. We propose student activities that illustrate what spatial curvature is and how it relates to the motion of planetary bodies in our Solar System.

1 Introduction

Embedding diagrams, particularly the one type that arises from the analysis of the equatorial plane motion in the Schwarzschild spacetime (see for example [1,2]) ,which we will also discuss here, have become the iconic image for General Relativity. Just a quick Internet image search using phrases such as *General Relativity* or *spacetime curvature* will show us hundreds of results of curved cone-like shapes, be they of the correct shape that is derived from GR or just artistic renditions inspired by the correct shape. Furthermore, a variety of T-shirts depicting this type of curved surface are commercially available and it has also been used in internationally popular TV shows such as The Simpsons. Therefore, we could argue that embedding diagrams are for GR in a similar position as the famous equation $E = mc^2$is for Special Relativity. Many people know of them or at least have seen them but do they understand them?

Because of their wide-spread appearance, we shouldn't ignore embedding diagrams even in our most basic exposition of GR. Although they allow us to visualize only spatial geometry (as opposed to *spacetime* geometry) and are therefore limited when it comes to explaining, for example, planetary motion, they help us discuss curvature, which is an essential concept in GR.

2 3D models

3D printing is a fast growing area of manufacture and thanks to the relatively low price of low-end 3D printers and especially thanks to the whole concept being open-source, 3D printers can now be found in many homes and schools even. With a 3D printer at hand, we can create plastic models that would be very complicated or even impossible to make otherwise.

Concerning GR, we can print curved surfaces, such as spheres, cones, embedding diagrams of the Schwarzschild spacetime with good accuracy and give them to students to introduce them to the concept of curvature. We don't have to rely on animations or inaccurate paper models (although in many cases that is a good place to start – as we will show).

A great advantage of 3D printing is that once we have made a computer model, we can print it as many times as we need only for the (relatively low) price the printing material and electricity. This brings the overall required time and cost of making the models substantially down.

3 Proposed activity

The main purpose of our activity is to introduce the concepts of *spatial curvature* and *geodesic motion.* We start with the idea from a book by Epstein [3], where a straight line is drawn on a piece of paper and the paper is then folded into a cone (see Fig. 1). We see our originally straight line being curved. Cone is actually, mathematically speaking, a first approximation to the embedding diagram in which we are interested. We can then use a proper (printed) cone (Fig. 2), to show that going “straight” on the one surface results in a curved trajectory. For this we have printed a small toy car, a sort of a steam roller, which can only move in a straight line. This we demonstrate on a flat surface, for example a desk. Provided the “steam roller” is sufficiently small compared to the curved surface, we can move it on the surface and its trajectory will be curved. It has no choice but to follow the intrinsic curvature of the surface. By attaching a marker to the “steam roller”, we can then draw the trajectory on our surfaces for better clarity.

We haven’t tried out this collection of activities yet, mainly because there is, unfortunately, no GR course at our department at the moment. We plan to organize a special voluntary workshop dedicated to this topic for future physics teachers studying at our department, as well as for a group of secondary-school students, by the end of this school year.

Fig. 1 Initial activity using paper cones

Fig. 2 3D-printed part for the treatment of spatial curvature

References

[1] Misner, C. W., Thorne, K. S., & Wheeler, J. A. (1973). *Gravitation*. San Francisco.

[2] Taylor, E. F., & Wheeler, J. A. (2000). *Exploring black holes introduction to general relativity*. San Francisco: Addison Wesley Longman.

[3] Epstein, L. C. (2000). *Relativity visualized*. San Francisco: Insight Press.