Graphs in Physics; Do They Correctly Depict Reality

Andrzej Sokolowski

Lone Star College, Mathematics and Physics Department, Houston 77385, TX, US

Abstract. Graphs in physics are essential to describe and encode information about phenomena. Graphs depicting a real motion also represent algebraic functions that must obey conditions for being such. A preliminary survey of physics resources has revealed inconsistencies in graph presenting. When given students to encode motion properties, such graph generated uncertainties and student confusion. The findings of this study call for a better alignment of physics and mathematics methods so that the students realize that physics supports the rules of mathematics.

1 Vertical Line Test as a Means to Justify Graphs for Functions

Functions of one variable representing real quantities assign a single and unique output for each of their inputs. As referred to graphs, this condition is supported by the Vertical Line Test (VLT), which states that "A curve in XY-plane is the graph of a function of *x*, if and only if no vertical line intersects the curve more than once" ([1] p. 17). The test assures that for each function input, $x=a$, only one and unique and real output is produced. The idea is visualized in Fig. 1. Graph A passes the VLT (the line in red) because it will assign a unique *y* value for each *x*-value when applied to the entire domain of $f(x)$. Graph B fails to pass the test at x_1 and x_2.

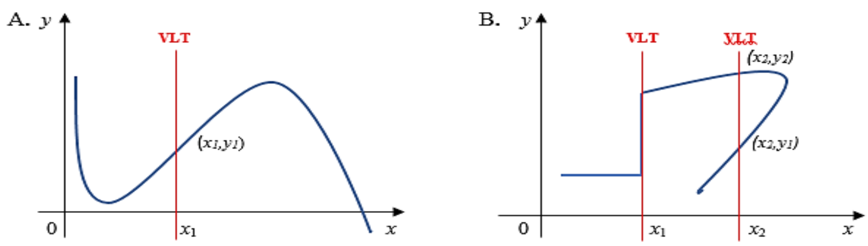

Fig.1 Application of the VLT to examine graphs for functions.

Why would the VLT be important in physics? When referred to kinematics, the VLT assures that object has a unique position, velocity or acceleration at each time instant. It also assures that the rates of changes of these functions' values can be computed. It is expected that graphs in physics produce real outputs on their domains. A preliminary survey of some physics textbooks has revealed that graphs, especially in kinematics that are to represent a real motion do not necessarily adhere to the principle of the VLT (see; [2], [3], [4], [5], [6], [7]).

2 Research Question, Data Collection, and Analysis

An ultimate query aroused how physics students interpret graphs that violate the VLT. This query generated the following research question: *Do graphs in physics need to adhere to rules of mathematics?* The following graph (Fig. 2), selected from a physics textbook [7] was supposed to model a real motion of a body. A sample of twenty-four high school physics students was given the graph and asked to find the velocity of the body at t = 9 s of motion along with justifying their answers. When scrutinized through the VLT, the graph does violate the VLT at 9th second. However, the students were not made aware of that flaw. The responses were clustered into two groups.

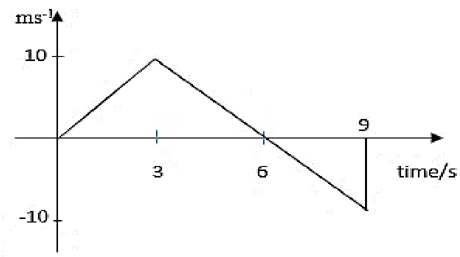

Fig. 2 Velocity - time graph

Group 1 (N=14) assumed that the graph represented a real motion. Within this group some students (N=10) claimed that the velocity is 0m/s, some (N=1) suggested -10m/s, and (N=3) a range of values from -10m/s to 0m/s. These students referred to scale on the vertical axis of the graph to support their answers. Group 2 (N=10) questioned the graph validity stating that the velocity is undefined. Selected justifications are, verbatim: *The graph needs a gradual change at t=9s, the graph is non-differential, the velocity cannot go perfectly up or down, the velocity cannot have multiple values at 9s, there is a sudden change at 9s.* While the students did not explicitly use the VLT to challenge the graph, they pointed out the flaw at 9s as producing unrealistic rapid changes of velocity, which in fact reflected the VLT outputs.

3 Discussion

Students who considered the data as reliable provided incorrect answers. These, perhaps with stronger algebraic skills, questioned the graph and supported their answers using correct algebraic justifications. Evaluating such responses is problematic though. Should the answers to Group 1 be graded as incorrect? Students most likely would admit the graph flaw if asked explicitly to justify the graph [8]? Moreover, it is hypothesized that giving students such graphs to analyze can also inadvertently question instructor's subject domain expertise.

This dilemma could be avoided if graphs in physics adhered to rules of algebra. Planinic at al. [9] concluded that students face difficulties with graph analysis in physics because, in addition to applying correct math tools, they need to interpret the results correctly. It seems that providing students with correct graphs would help to overcome this difficulty.

References

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