Blending of mathematics and physics: student difficulties with boundary conditions for the diffusion equation

Sofie VAN DEN EYNDE[1,3], Martin GOEDHART[1], Johan DEPREZ[2] and Mieke DE COCK[3]
[1]*Rijksuniversiteit Groningen, ISEC, Nijenborgh 9, 9747 AG Groningen, the Netherlands*
[2]*KU Leuven, Department of Mathematics & LESEC, Celestijnenlaan 200b box 2400, 3001 Leuven, Belgium.*
[3]*KU Leuven, Department of Physics and Astronomy & LESEC, Celestijnenlaan 200c – box 2406, 3001 Leuven, Belgium.*

Abstract. This study reports on student difficulties in connecting mathematics and physics in the context of the diffusion equation. To study how students blend both fields in the context of partial differential equations, we administered task-based interviews with 12 students from two universities. We looked at student answers through the lens of conceptual blending, and analysis of these interviews revealed a number of difficulties that were categorized using the blending framework. We thereby found difficulties can be situated both in the mathematics and physics space, but also in the blended space, were they come together.

1 Introduction and problem statement

The role of mathematics in physics is an important topic in physics education research. The intrinsic mathematical nature of physics implies that understanding in physics is strongly connected to the ability to think about the world in mathematical structures. As such, proficiency in mathematics is required to understand physical phenomena and being able to combine the two fields is a prerequisite to become proficient in physics. The diffusion equation is a beautiful example where mathematics and physics come together. To study how students blend mathematics and physics, we therefore chose the context of Partial Differential Equations (PDEs), more specifically the diffusion/heat equation.

While investigating students' blending process in the context of the diffusion equation, we observed many student difficulties both of mathematical and physical nature, but also in blending the two. In this presentation, we will give an overview of student difficulties related to understanding boundary conditions (BCs) for the diffusion equation and give some implications for instruction.

2 Methodology and theoretical framework

To study student reasoning while solving problems on the heat/diffusion equation, we conducted task-based interviews with undergraduate students at two universities: KU Leuven (Belgium) (N=6) and the University of Groningen (the Netherlands) (N=6). All students encountered PDEs in one of their courses.

Interview tasks were developed with particular attention to problems where mathematical and physics come together to solve the task: interpretation of the equation, interpretation of the solution and mathematical formulation of the boundary conditions when the physical system is described verbally. All interviews were audio- and videotaped using a smart-pen and verbally transcribed. Analysis was done using the lens of the conceptual blending framework. Conceptual blending, sometimes called mental space integration, is a framework introduced by Fauconnier and Turner (1) to model how people create new meaning by selectively combining information garnered from previous experiences and is recently used in the context of PER by different authors (2–4).

A mental space is comprised of conceptual packets or knowledge elements that tend to be activated together, and has an organizing frame that specifies the relationships among the elements within the mental space. According to the conceptual blending framework, two or more input mental spaces that share content or structure can be combined into a new, blended space.

In this contribution, we report on the findings of a subtask in which students had to set up a mathematical description of the boundary conditions for a given physical situation:

> *"In a tube with a length of one meter there are u_0 particles. At time $t = 0$ they are distributed as the function $f(x) = u_0(1 - \cos 2\pi x)$. The left and right end of the tube are closed so no particles can flow in or out of the tube. Write down the mathematical description of this physical situation (PDE, boundary and initial conditions). Also make a sketch of the initial distribution of the particles."*

3 Results and discussion

During the interviews, we observed many difficulties with BCs that prevent students from blending or lead to an incorrect blend. A first group of difficulties can be situated in the mathematics input space. Students have difficulties with the concept of BCs, what their meaning is, which form they can have and what their role is in solving a PDE. There are several difficulties with functions of two variables: students do not specify with respect to which variable a derivative is taken, they have difficulties interpreting these different partial derivatives or they cannot distinguish between them. Some students confuse BCs with the initial condition, which is partly connected to difficulties with two variables.

A second group of difficulties can be situated in the physics input space. Some students have difficulties with imagining the physical situation and specifically with determining the physical boundaries of the system and almost all students have difficulties with the concept 'isolated boundaries'.

This difficulty returns in the blended space, where students had difficulties connecting mathematical concepts and descriptions to that idea of 'isolated boundaries'. In general, many blends fail because the input spaces contain incorrect or incomplete elements which students bring together incorrectly during blending.

Getting insight in difficulties that prevent students to blend physics and mathematical ideas productively will allow to specifically address this issue in instruction.

References

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